Deep Learning (1470)

Randall Balestriero

Class 5: Optimization and Hyper-parameters

Rewind

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- How to search for the right “architecture”?

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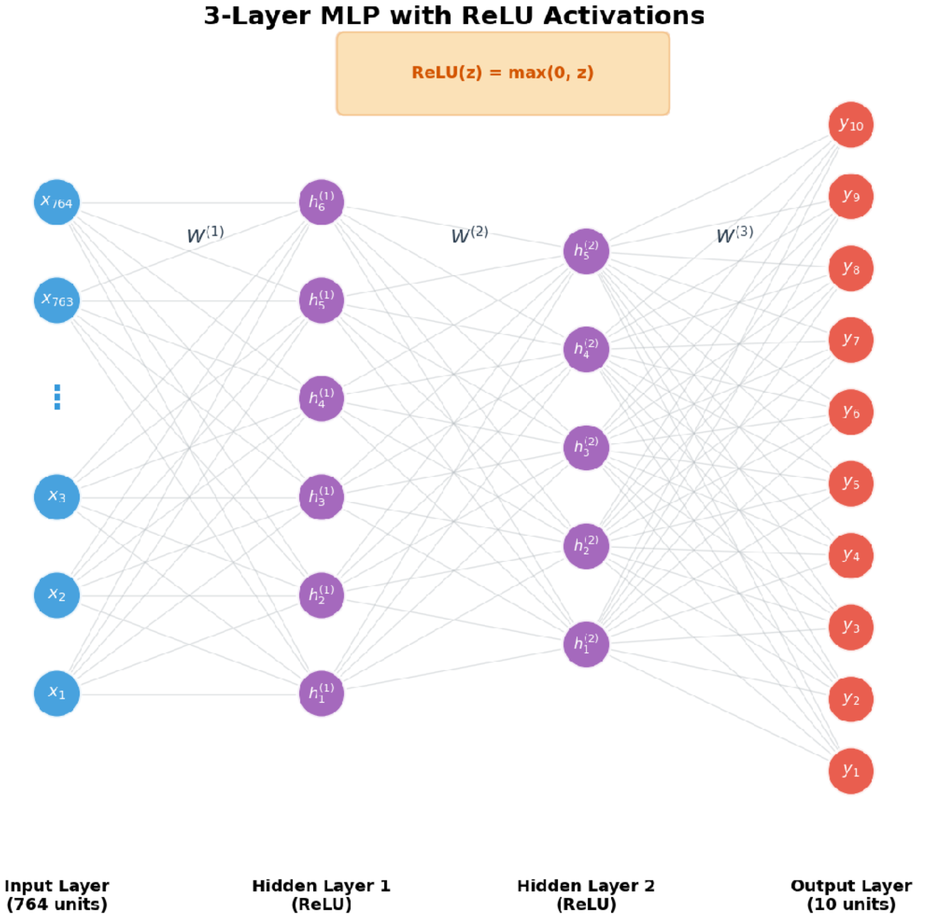
- How to search for the right “architecture”?
- How many lines of codes to implement the MNIST model and reach 99.5%?

Your First Deep Network!



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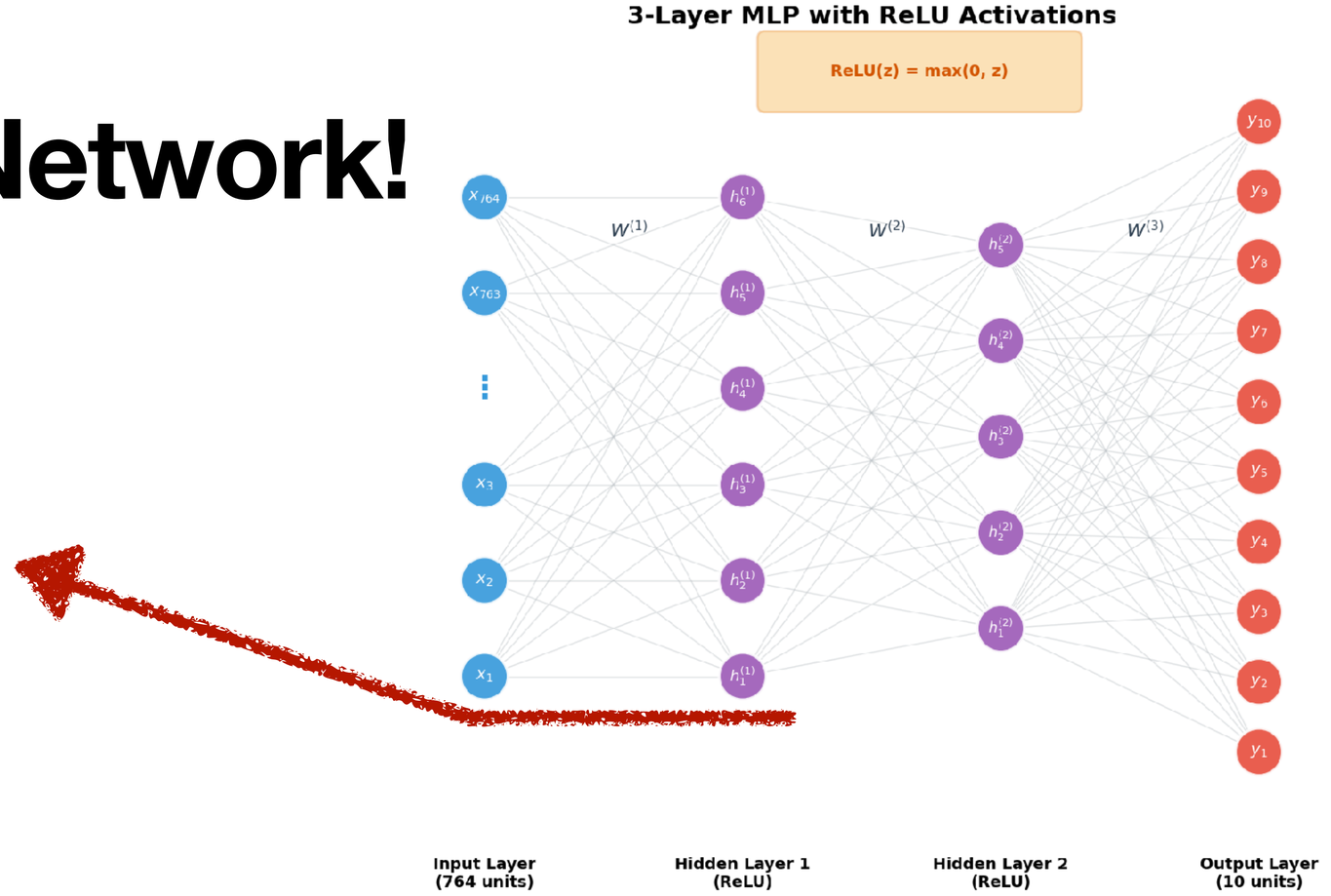

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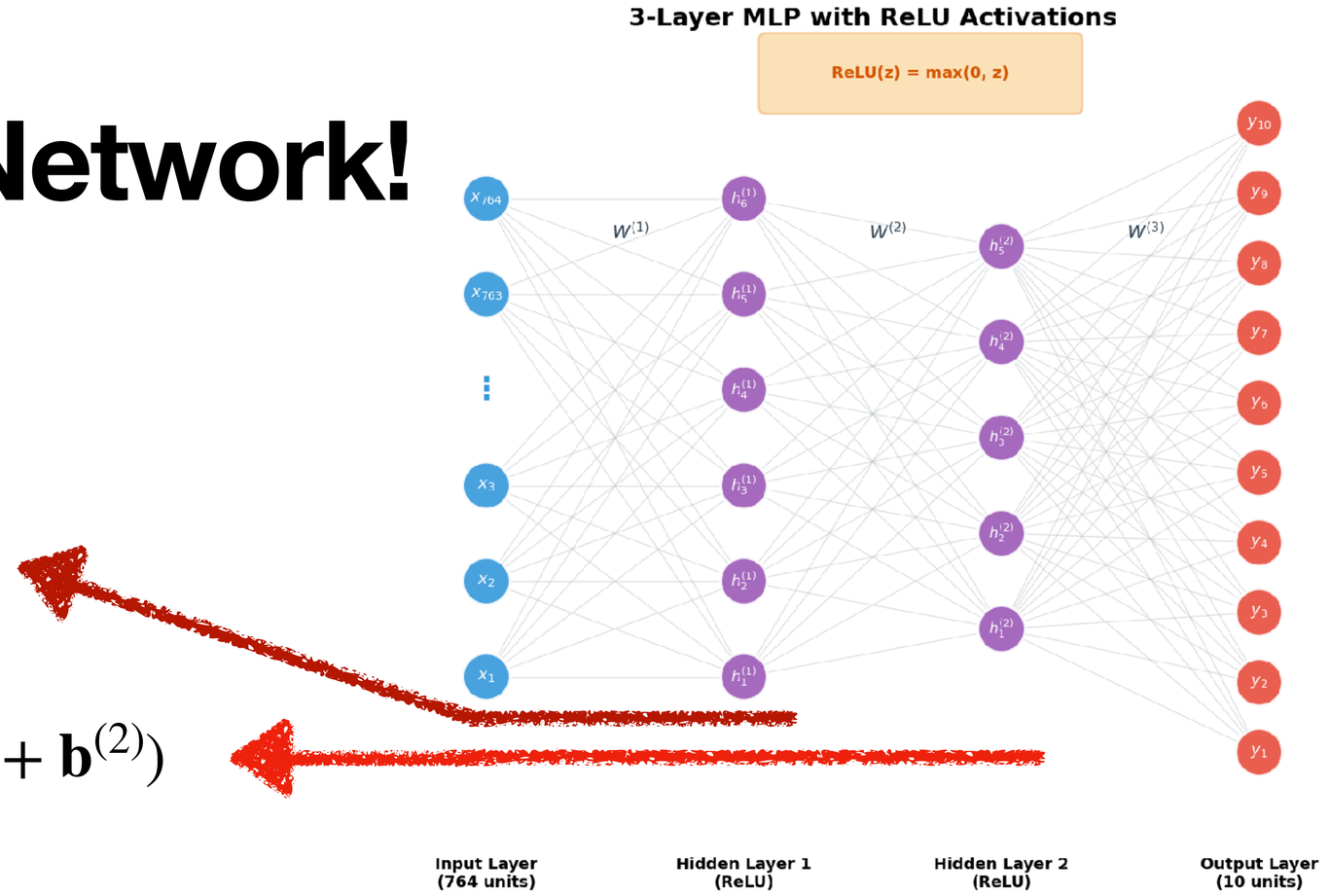

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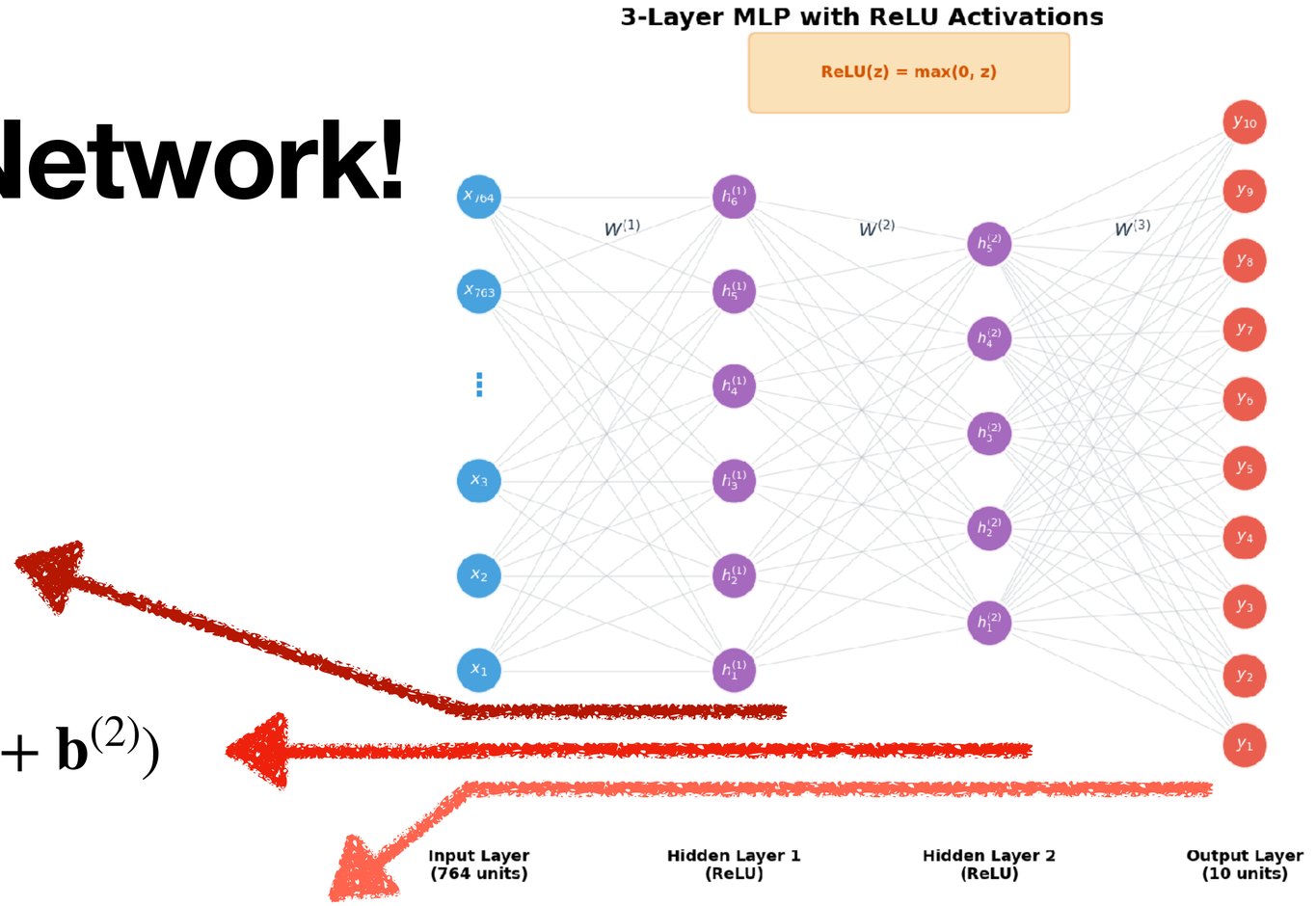

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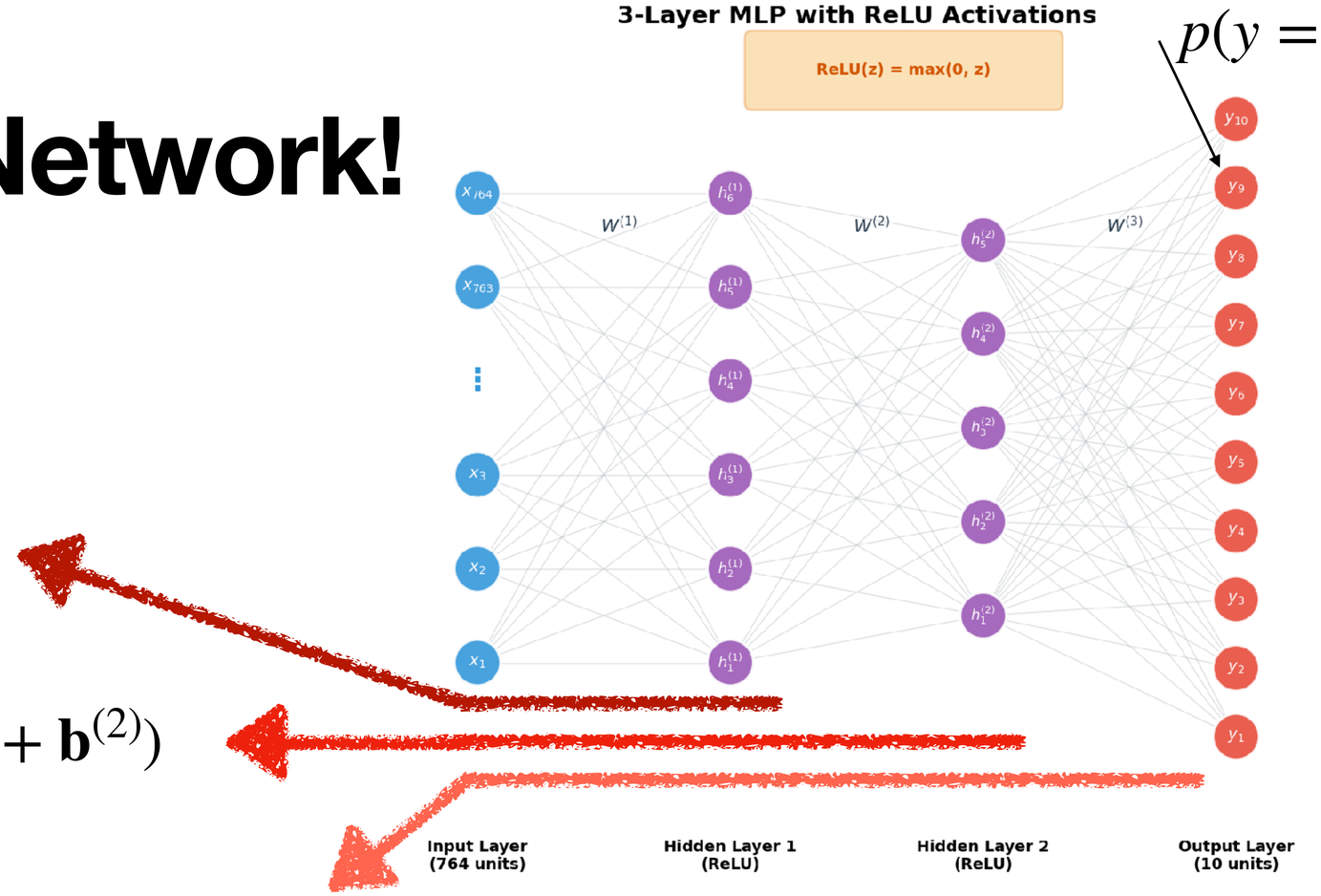

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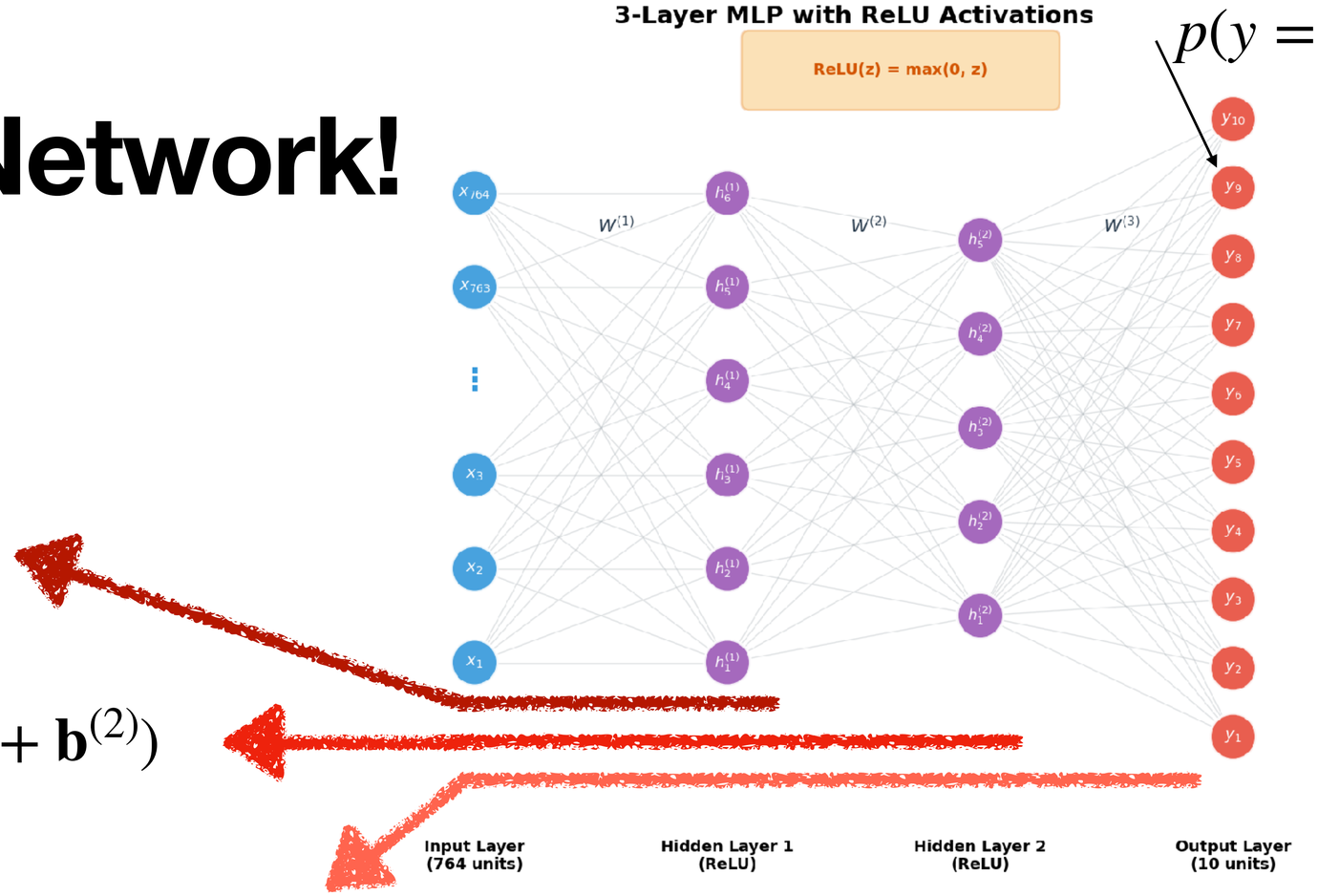

How to get p(y=9|x_n)?

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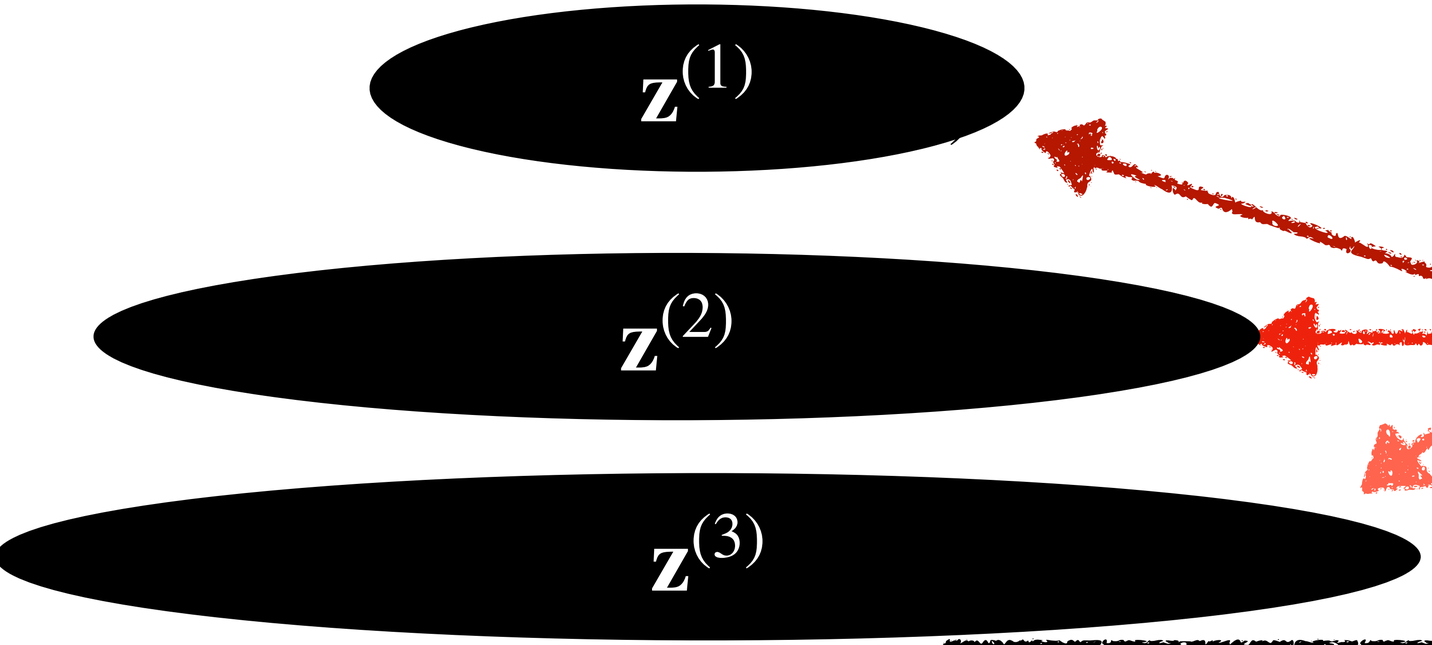

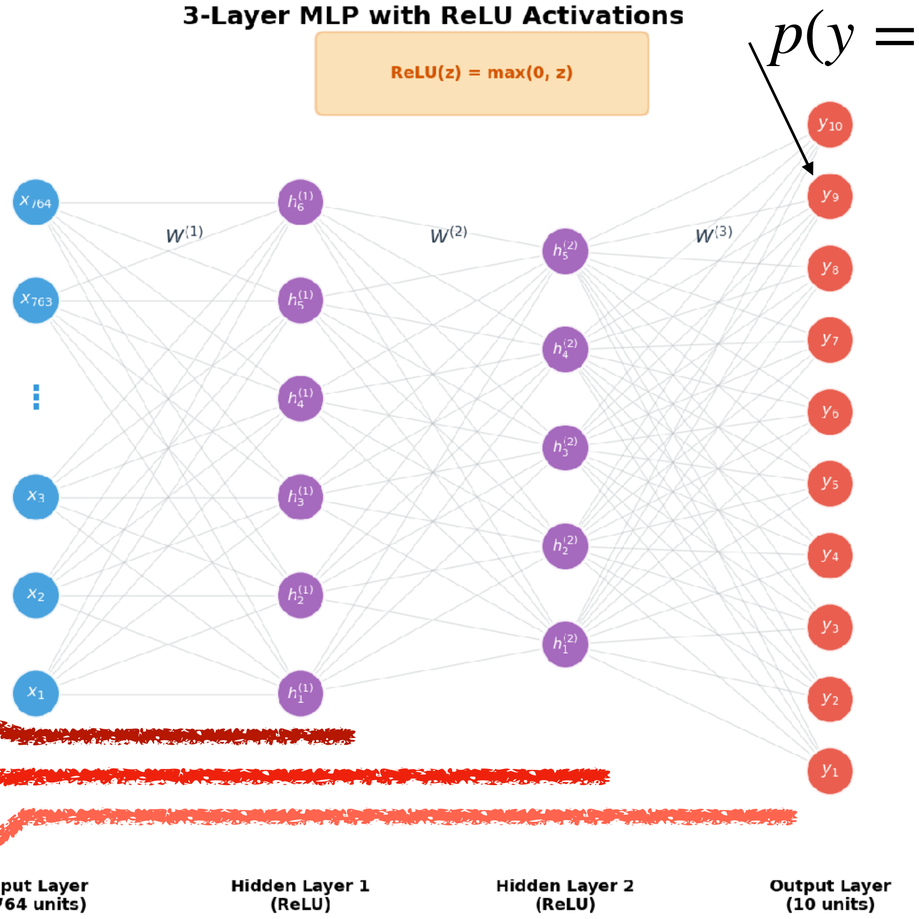

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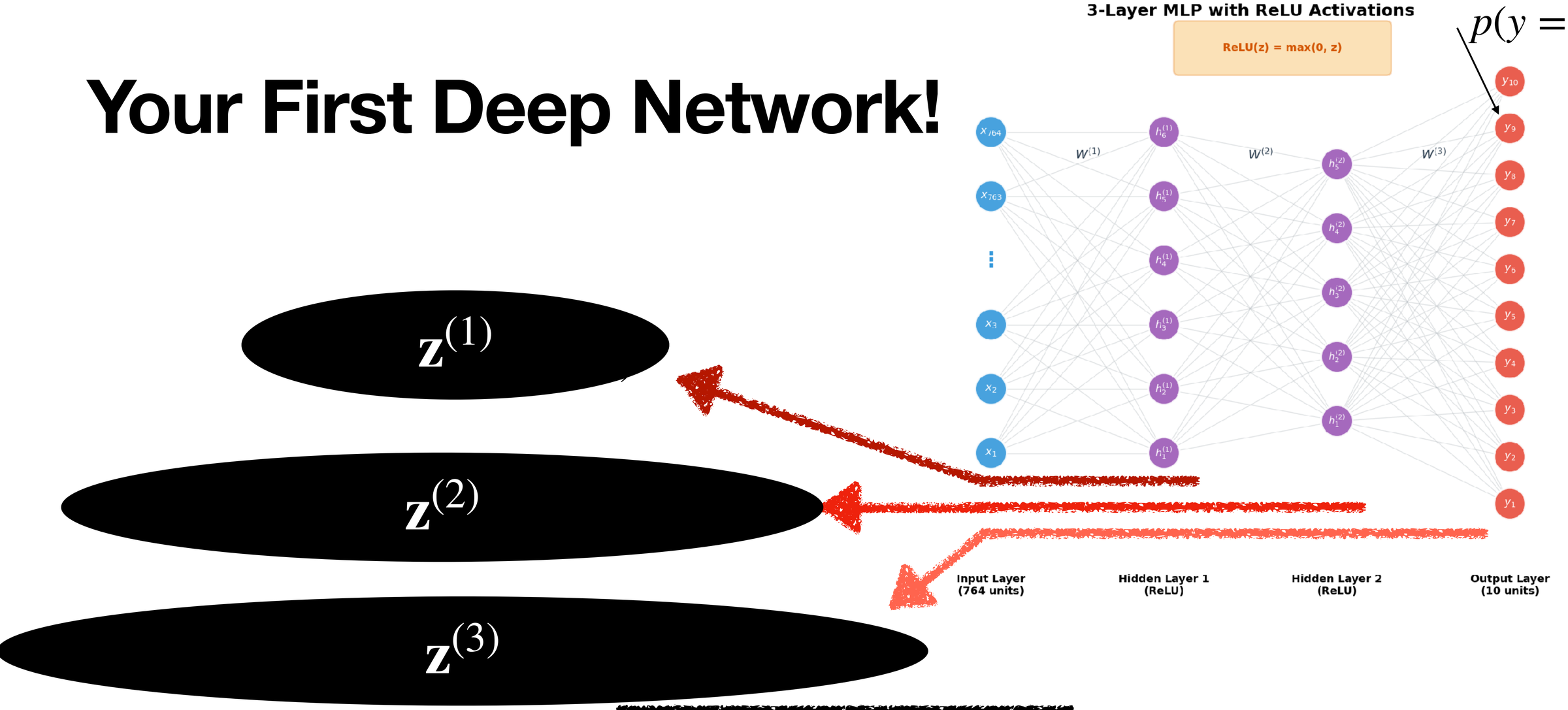

$\mathbf{z}^{(1)}$

$\mathbf{z}^{(2)}$

$\mathbf{z}^{(3)}$

How to get p(y=9|x_n)?

$$p(y = c \mid \mathbf{x}_n) = \text{Softmax}(\mathbf{z}^{(3)})_c$$

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- Think of it as taking any real vector and turning it into a probability vector

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<stdin>:2: RuntimeWarning: overflow encountered in exp
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$$= \sum_{n=1}^{N} - \mathbf{z}^{(3)}(\mathbf{x}_n)_{y_n} + \log(\sum_{c=1}^{C} e^{\mathbf{z}^{(3)}(\mathbf{x}_n)_c})$$

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Fused op (cross_entropy + softmax)!!

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- Take the gradient of the loss w.r.t. $\mathbf{z}^{(3)}$, then multiply by the gradient of $\mathbf{z}^{(3)}$ w.r.t. $\mathbf{W}^{(3)}$ and so on!

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- What is the gradient of $\mathbf{z}^{(2)}$ w.r.t. $\mathbf{W}^{(2)}$? $\mathbf{D}_n^{(2)}$
- What is the gradient of the loss w.r.t. $\mathbf{W}^{(2)}$? $(\mathbf{v}_n^\top \mathbf{W}^{(3)} \mathbf{D}_n^{(2)})^\top \mathbf{z}^{(1)}(\mathbf{x}_n)$

The Full Story

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- Compute the loss from your samples $\{(\mathbf{x}_n, \mathbf{y}_n), n = 1, \ldots, N\}$

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Can you think of an improvement?

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How to pick the mini-batch size?

Beyond Gradient Descent

Optimizer	Update idea (per step)	Strengths	Weaknesses / Notes
SGD	`θ ← θ − lr ∗ g`	Simple, stable; often good generalization	Requires tuning `lr`; can be slow without momentum
SGD + Momentum	`v ← μv + g`; `θ ← θ − lr ∗ v`	Faster convergence; smooths noisy gradients	Extra hyperparameter (momentum μ)
SGD + Nesterov	Look ahead before gradient step	Often converges faster than vanilla momentum	Slightly more complex; similar tuning issues
Adam	Per-parameter adaptive lr using 1st & 2nd moment estimates	Usually works "out of the box"; good for sparse grads	Can generalize worse than SGD; more hyperparameters
AdamW	Adam + *decoupled* weight decay	Better weight decay behavior; often preferred default	Slightly more to configure (weight_decay)
RMSprop	Scales lr by running avg of squared grads	Good for non-stationary problems; common in RNNs	Can be sensitive to hyperparameters
Adagrad	Accumulates squared grads, shrinking lr over time	Good for sparse features; no manual lr schedule	Learning rate can become too small over long training

The Full (real) Story

```
for x_batch, y_batch in loader:
    optimizer.zero_grad()
    # Forward pass: raw logits (no softmax)
    logits = model(x_batch)
    # Functional cross entropy:
    # takes logits and class indices directly
    loss = F.cross_entropy(logits, y_batch)
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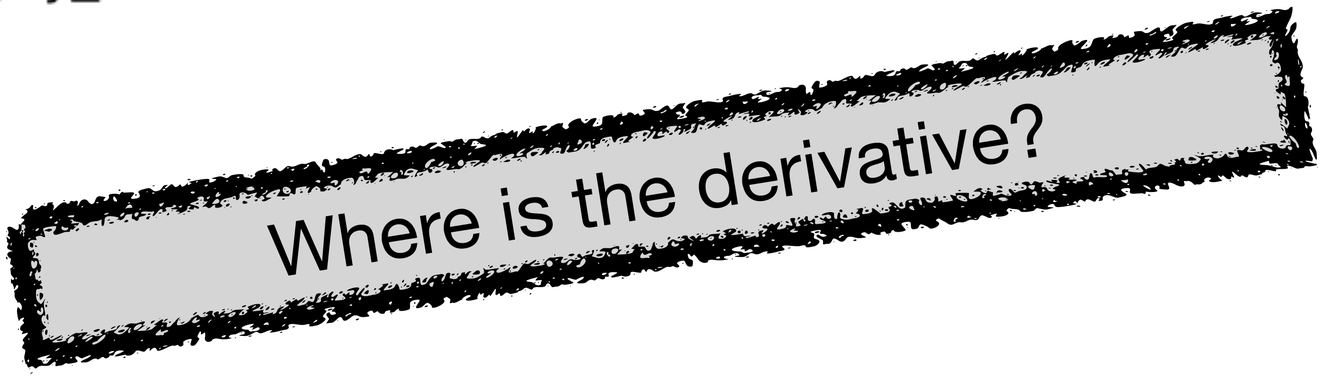

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- The efficient implementation is called “backdrop” with vjp (or jvp)

The Elephant in the Room

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- Which framework to use?

The Elephant in the Room

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What are our hyper-parameters so far?

- Preprocessing of the data
- Number of layers (L), width of each layer
- Initialization of the parameters
- Optimizer, learning rate (+ momentum, …)
- Mini-batch size
- Training steps

Initialization Brainstorming

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Initialization Brainstorming

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What would be a good/bad initialization?