Deep Learning

Day 20: Policy Gradient and Actor Critic

CSCI 1470

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Thursday, 11/13/25

Terminology Review

MDP: Markov Decision Process

<States, Actions, Reward Function, Transition Probabilities, Discount Factor>

Episode: Single run-through of an MDP (from start to terminal state)

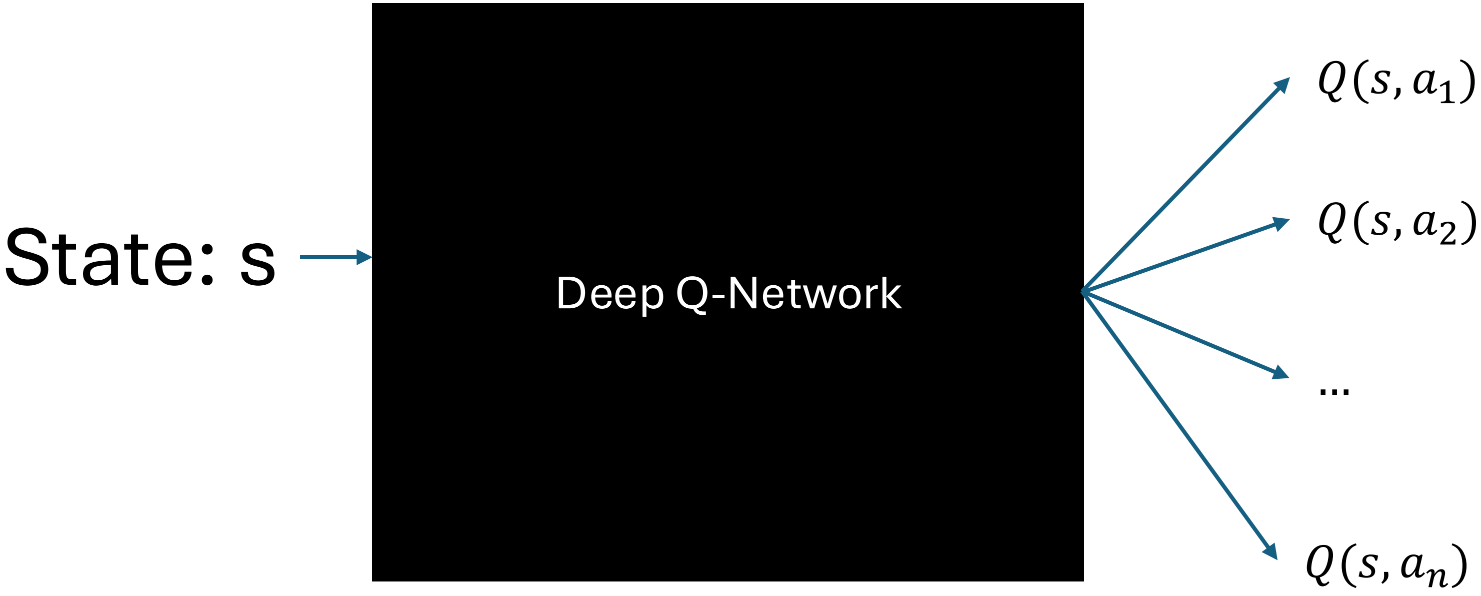
Trajectory: states, actions, and rewards received in an episode

Returns: Total (discounted) rewards of a trajectory

Value: Expected Returns from a specific state

Q-Value: Expected Returns from a specific state when taking a specific action

Deep-Q Network



Deep Q-Networks (DQNs):

1. Take in a state
2. Return Q-values for each action

Deep-Q Learning

Initialize DQN to approximate Q

Maintain estimates of Q(s, a) for all (s, a) pairs

Collect experiences, update Q estimates with:

Compute $L_\theta = \left[r + \gamma \max_{a'} Q_\theta(s', a') \ - Q_\theta(s, a)\right]^2$

update θ with SGD on Loss function

Gymnasium

Gymnasium

Provides a common implementation of MDPs for RL.

A gym environment has a current state that can be updated by calling environment.step(action)

```python
import gymnasium as gym

# Initialise the environment
env = gym.make("LunarLander-v3", render_mode="human")

# Reset the environment to generate the first observation
observation, info = env.reset(seed=42)
for _ in range(1000):
    # this is where you would insert your policy
    action = env.action_space.sample()

    # step (transition) through the environment with the action
    # receiving the next observation, reward and if the episode has terminated or truncated
    observation, reward, terminated, truncated, info = env.step(action)

    # If the episode has ended then we can reset to start a new episode
    if terminated or truncated:
        observation, info = env.reset()

env.close()
```

Implementing DQNs

```
# Compute Q(s_t, a) - the model computes Q(s_t), then we select the
# columns of actions taken. These are the actions which would've been taken
# for each batch state according to policy_net
state_action_values = policy_net(state_batch).gather(1, action_batch)

# Compute V(s_{t+1}) for all next states.
# Expected values of actions for non_final_next_states are computed based
# on the "older" target_net; selecting their best reward with max(1).values
# This is merged based on the mask, such that we'll have either the expected
# state value or 0 in case the state was final.
next_state_values = torch.zeros(BATCH_SIZE, device=device)
with torch.no_grad():
    next_state_values[non_final_mask] = target_net(non_final_next_states).max(1).values
# Compute the expected Q values
expected_state_action_values = (next_state_values * GAMMA) + reward_batch

# Compute Huber loss
criterion = nn.SmoothL1Loss()
loss = criterion(state_action_values, expected_state_action_values.unsqueeze(1))
```

Source: Torch DQN tutorial https://docs.pytorch.org/tutorials/intermediate/reinforcement_q_learning.html

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What should the activation function be for the final layer?

How do we train a policy network?

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$$\pi = \mathrm{argmax}_{\pi}\,(V(s_0))$$

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What's our objective?

Find a policy π such that the value of the start state is maximized:

$$\pi = \text{argmax}_{\pi} \left(V(s_0)\right)$$

How can we maximize $V(s_0)$?

Let $J(\theta)$ be our objective function:

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Probability of a trajectory occurring

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Probability of taking an action for a given state

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$$J(\theta) = \sum_{\tau} \Pr(\tau|\theta)\, G(\tau)$$

Sum over all possible trajectories

Probability of a trajectory occurring

Returns of a specific trajectory

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State transition Probability

Probability of taking an action for a given state

Log-Derivative Trick

We can rewrite the derivative of a function using the derivative of the natural log function:

$$\nabla \ln f(x) = \frac{\nabla \mathrm{f}(x)}{f(x)}$$

$$\nabla f(x) = f(x) \nabla \ln \mathrm{f}(\mathrm{x})$$

When applied to $\Pr(\tau|\theta)$:

$$\nabla_\theta \Pr(\tau|\theta) = \Pr(\tau|\theta)\, \nabla_\theta \ln \Pr(\tau|\theta)$$

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$$\nabla_\theta \ln \Pr(\tau|\theta) = \nabla_\theta \sum_{t=0}^{T} \ln P(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

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Log of product -> sum of logs

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Derivative of sum -> sum of derivative

$$\nabla_\theta \ln \Pr(\tau|\theta) = \sum_{t=0}^{T} \nabla_\theta \ln P(s_{t+1}|s_t, a_t) + \nabla_\theta \ln \pi_\theta(a_t|s_t)$$

Gradient of a trajectory

$$\nabla_\theta \ln \Pr(\tau|\theta) = \sum_{t=0}^{T} \nabla_\theta \ln P(s_{t+1}|s_t, a_t) + \nabla_\theta \ln \pi_\theta(a_t|s_t)$$

State transition function
does not depend on θ!

$$\nabla_\theta \ln \Pr(\tau|\theta) = \sum_{t=0}^{T} \nabla_\theta \ln \pi_\theta(a_t|s_t)$$

Policy Gradient Derivation

Putting it all back together:

$$J(\theta) = \sum_{\tau} \Pr(\tau|\theta)\, G(\tau)$$

Our Objective

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$$\nabla_\theta J(\theta) = \sum_{\tau} \Pr(\tau|\theta) G(\tau) \nabla_\theta \ln \Pr(\tau|\theta)$$

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$$J(\theta) = \sum_{\tau} \Pr(\tau|\theta)\, G(\tau)$$ Our Objective

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$$\nabla_\theta J(\theta) = \sum_{\tau} \left[\Pr(\tau|\theta) G(\tau) \sum_{t=0}^{T} \nabla_\theta \ln \pi_\theta(a_t|s_t)\ \right]$$ Gradient of a Trajectory

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$$\nabla_\theta J(\theta) = \mathbb{E}[G_0 \sum_{t=0}^{T} \nabla_\theta \ln \pi_\theta(a_t|s_t)]$$ Convert back to Expectation

Policy Gradient

Bigger step if better returns

Direction to move in to increase probability of trajectory

$$\nabla_\theta J(\theta) = \mathbb{E}[G_0 \sum_{t=0}^{T} \nabla_\theta \ln \pi_\theta(a_t|s_t)]$$

We will never be able to sum over all possible trajectories…

How do we get around this?

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How do we get around this?

Sampling!

1. Collect n trajectories following policy π_θ
2. $\Pr(\tau|\theta) = 1/n$ for each trajectory
3. Calculate the total return for each trajectory $G(\tau)$

Reward-To-Go Policy Gradient

You can also do the policy gradient derivation such that the gradient does not depend on G_0, but on G_t

$$\nabla_\theta J(\theta) = \mathbb{E}[\sum_{t=0}^{T} G_t \nabla_\theta \ln \pi_\theta(a_t|s_t)]$$

Or

$$\nabla_\theta J(\theta) = \mathbb{E}[\sum_{t=0}^{T} Q(s_t, a_t) \nabla_\theta \ln \pi_\theta(a_t|s_t)]$$

REINFORCE (Policy Gradient Learning)

REINFORCE, A Monte-Carlo Policy-Gradient Method (episodic)

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$
Repeat forever:
    Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
    For each step of the episode $t = 0, \ldots, T-1$:
        $G \leftarrow$ return from step t
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G \nabla_{\boldsymbol{\theta}} \ln \pi(A_t|S_t, \boldsymbol{\theta})$

Source: Sutton and Barto, Reinforcement Learning: An Introduction

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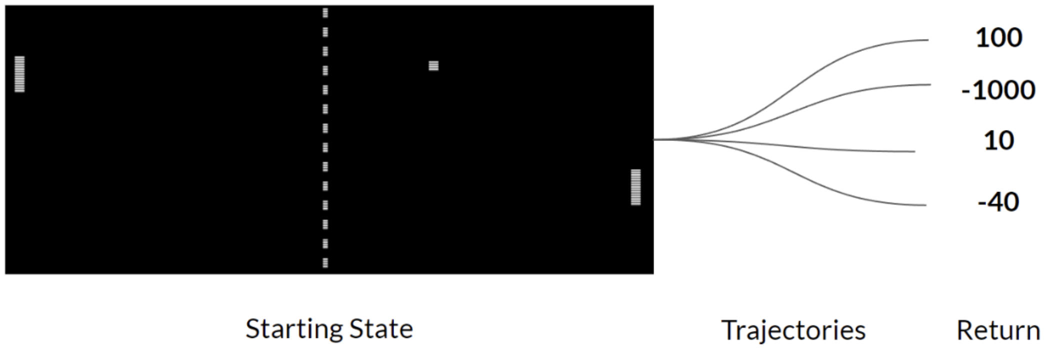
Why is the update adding the gradient instead of subtracting?

When π is based on a softmax, $\nabla_\theta \ln \pi_\theta(a|s)$ is actually easy to compute by hand using log rules and the fact that $\ln e^x = x$

Source: Sutton and Barto, Reinforcement Learning: An Introduction

Variance of REINFORCE



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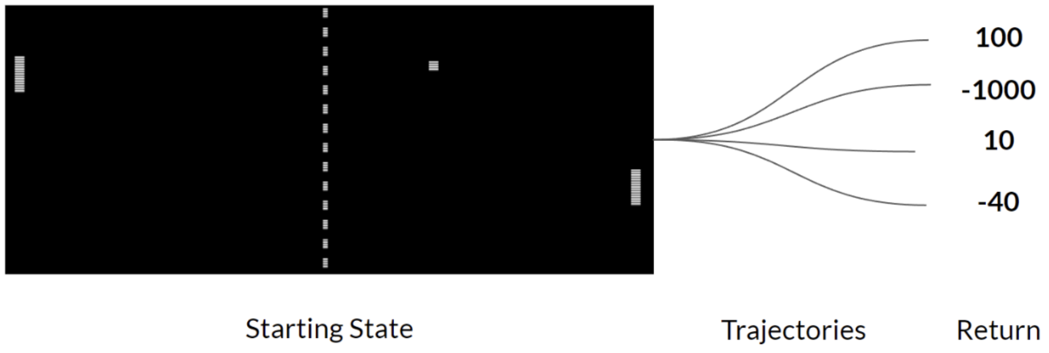

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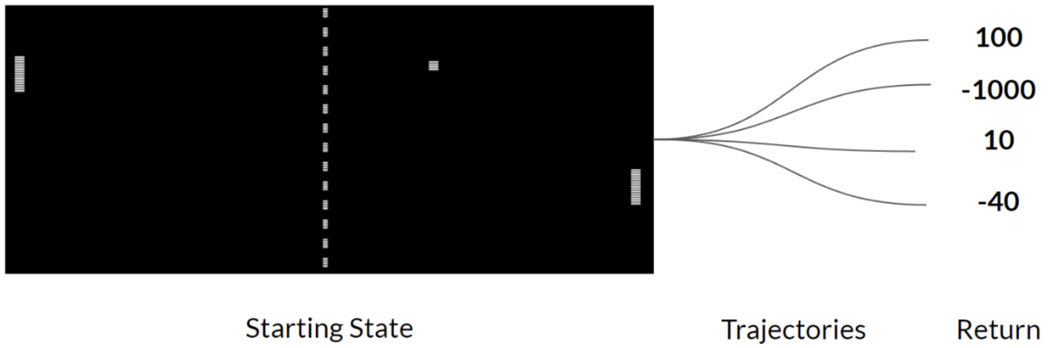

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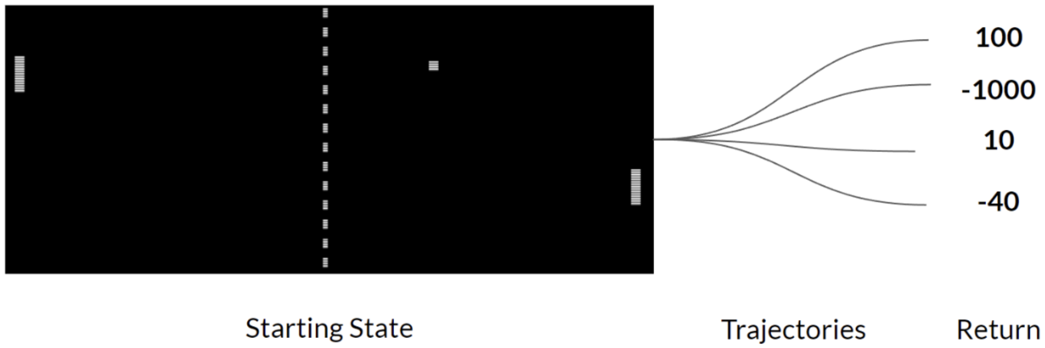

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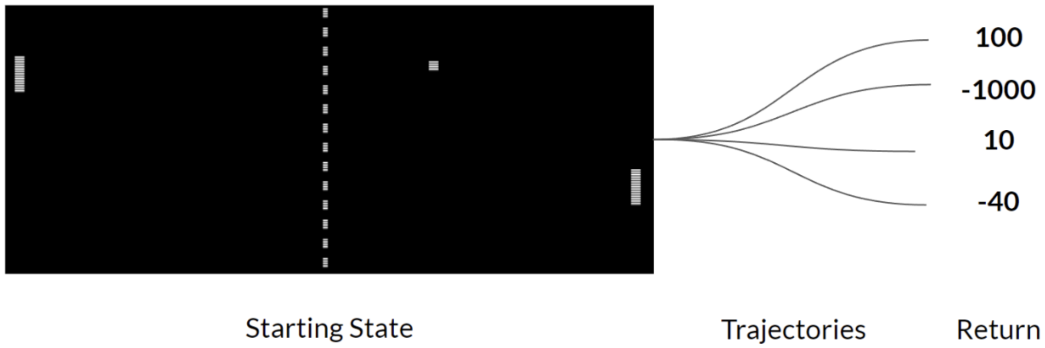

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Baseline functions can reduce the variance of the gradient estimate

The value function V(s) is the ideal baseline function

Derivation of REINFORCE w/ Baseline Function

$$\nabla_\theta J(\theta) = \mathbb{E}[\sum_{t=0}^{T} (G_t - b(s))\, \nabla_\theta \ln \pi_\theta(a_t|s_t)]$$

First, let's show that the gradient estimate is unbiased. We see that with the baseline, we can distribute and rearrange and get:

$$\nabla_\theta \mathbb{E}_{\tau\sim\pi_\theta}[R(\tau)] = \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) \left(\sum_{t'=t}^{T-1} r_{t'}\right) - \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) b(s_t)\right]$$

Due to linearity of expectation, all we need to show is that for any single time t, the gradient of $\log \pi_\theta(a_t|s_t)$ multiplied with $b(s_t)$ is zero. This is true because

$$\begin{aligned}
\mathbb{E}_{\tau\sim\pi_\theta}\Big[\nabla_\theta \log \pi_\theta(a_t|s_t) b(s_t)\Big] &= \mathbb{E}_{s_{0:t},a_{0:t-1}}\Big[\mathbb{E}_{s_{t+1:T},a_{t:T-1}}[\nabla_\theta \log \pi_\theta(a_t|s_t) b(s_t)]\Big] \\
&= \mathbb{E}_{s_{0:t},a_{0:t-1}}\Big[b(s_t)\cdot \underbrace{\mathbb{E}_{s_{t+1:T},a_{t:T-1}}[\nabla_\theta \log \pi_\theta(a_t|s_t)]}_{E}\Big] \\
&= \mathbb{E}_{s_{0:t},a_{0:t-1}}\Big[b(s_t)\cdot \mathbb{E}_{a_t}[\nabla_\theta \log \pi_\theta(a_t|s_t)]\Big] \\
&= \mathbb{E}_{s_{0:t},a_{0:t-1}}\Big[b(s_t)\cdot 0\Big] = 0
\end{aligned}$$

$$\mathbb{E}_{a_t}\Big[\nabla_\theta \log \pi_\theta(a_t|s_t)\Big] = \int \frac{\nabla_\theta \pi_\theta(a_t|s_t)}{\pi_\theta(a_t|s_t)} \pi_\theta(a_t|s_t) da_t = \nabla_\theta \int \pi_\theta(a_t|s_t) da_t = \nabla_\theta \cdot 1 = 0$$

Derivation: https://danieltakeshi.github.io/2017/03/28/going-deeper-into-reinforcement-learning-fundamentals-of-policy-gradients/

REINFORCE with Baseline

REINFORCE with Baseline (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Algorithm parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
    $\delta \leftarrow G - \hat{v}(S_t,\mathbf{w})$
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S_t,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

Pseudocode uses SGD, but you can just as easily use any other optimizer (e.g., Adam)

Source: Sutton and Barto Chapter 13

REINFORCE with Baseline

REINFORCE with Baseline (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Algorithm parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)

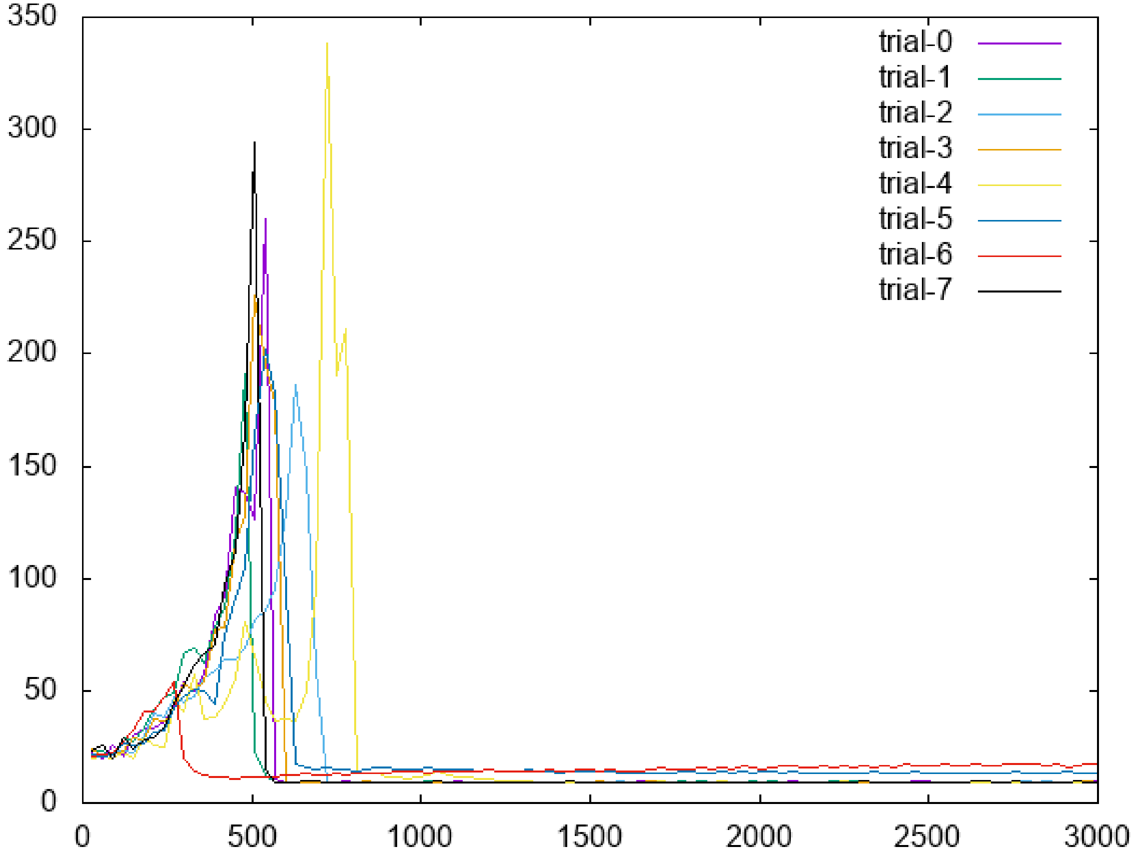
Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
    $\delta \leftarrow G - \hat{v}(S_t,\mathbf{w})$
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \boxed{\delta \nabla \hat{v}(S_t,\mathbf{w})}$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

Gradient of L $= \frac{1}{2}\delta$^2

Pseudocode uses SGD, but you can just as easily use any other optimizer (e.g., Adam)

Source: Sutton and Barto Chapter 13

Key Idea for RL: Variance is the enemy



Cartpole

Policy Collapse: https://stats.stackexchange.com/questions/252685/policy-gradient-reward-collapse

Let’s do an example with Multi-Arm Bandits

What’s a one-armed bandit?

Multi-Arm Bandits

Multi-Arm Bandits

Single-armed bandit

Multi-Arm Bandits

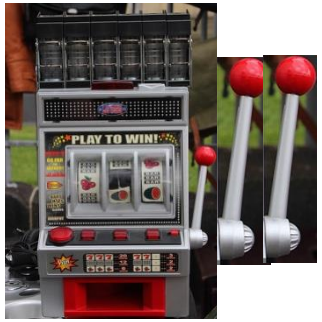
Single-armed bandit

Multi-Arm Bandits

Single-armed bandit

Multi-Arm Bandits

Single-armed bandit



Multi-Arm Bandits

Single-armed bandit

Multi-Arm Bandits

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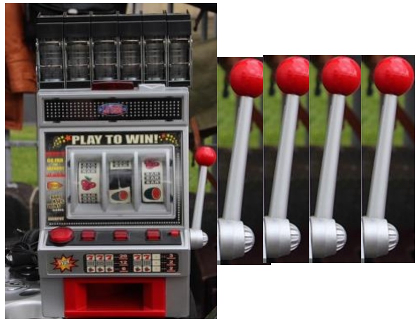

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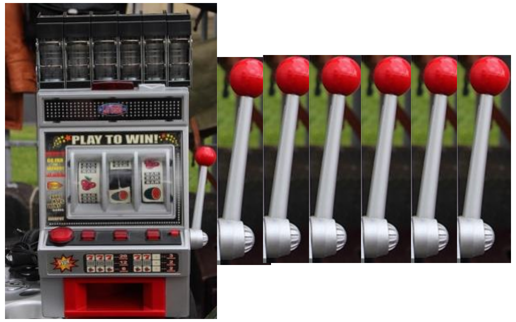

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When an arm is pulled, the rewards are random.

Each arm returns a reward with (different) unknown mean and variance

Single-armed bandit

Multi-Arm Bandits

When an arm is pulled, the rewards are random.

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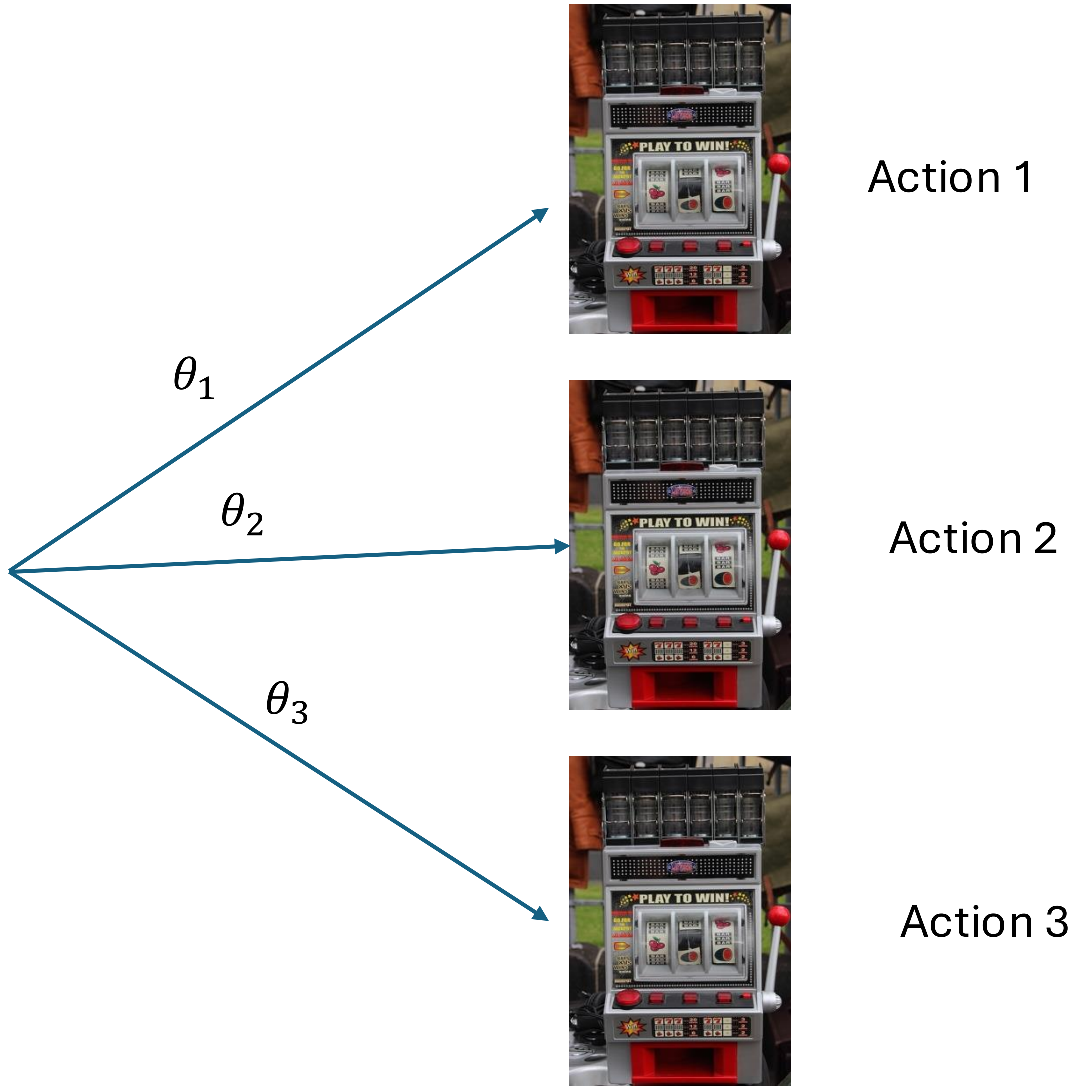
Single-armed bandit

Bandit Problems are essentially MDPs with a single state.

Useful testbed for a number of algorithms and very useful for theory

Policy Gradient on Multi-Arm Bandits

Maintain parameter for each action, θ_i

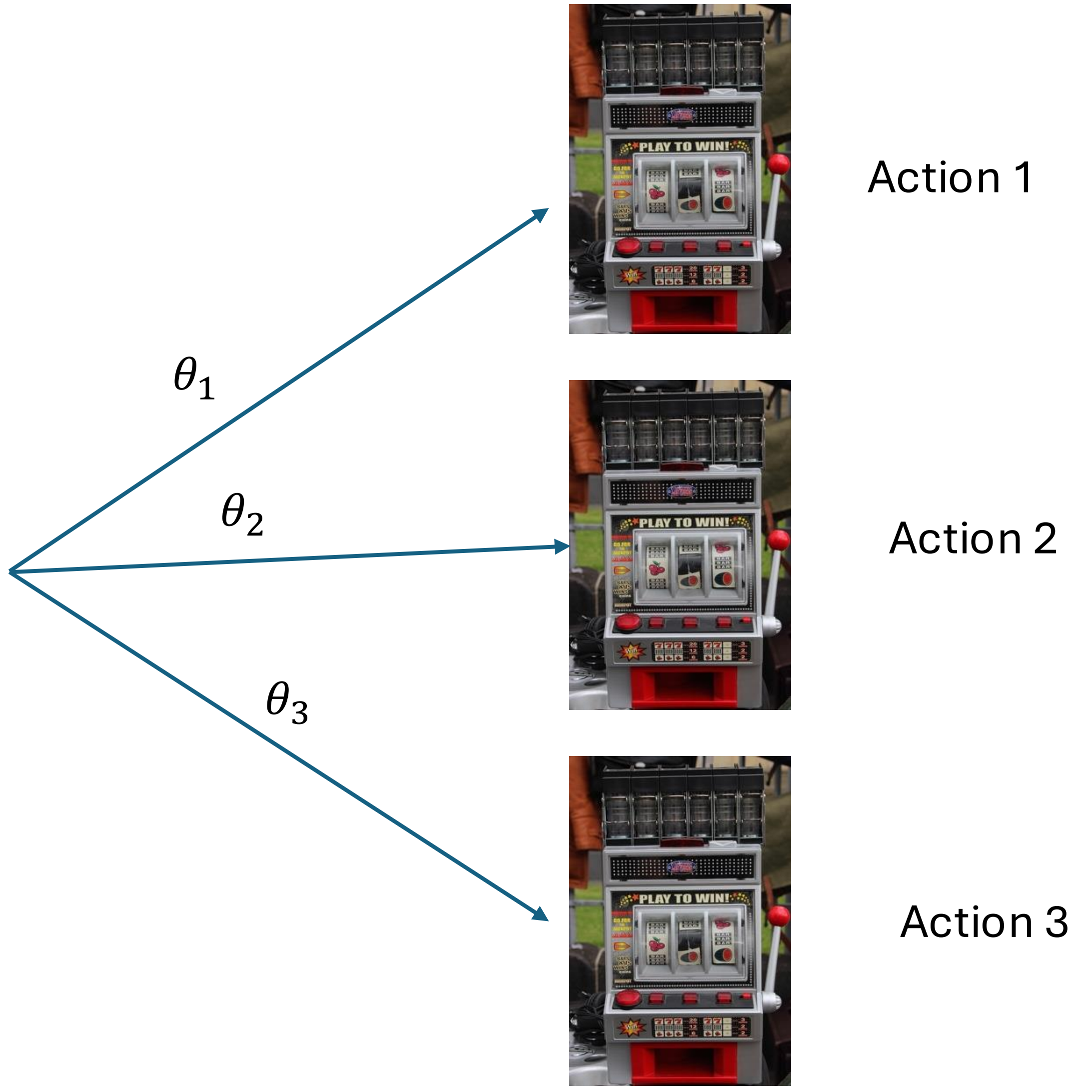


Policy Gradient on Multi-Arm Bandits

Maintain parameter for each action, θ_i

Take Action according to Softmax:

$$\pi_\theta(a_1) = \frac{e^{\theta_1}}{e^{\theta_1} + e^{\theta_2} + e^{\theta_3}}$$



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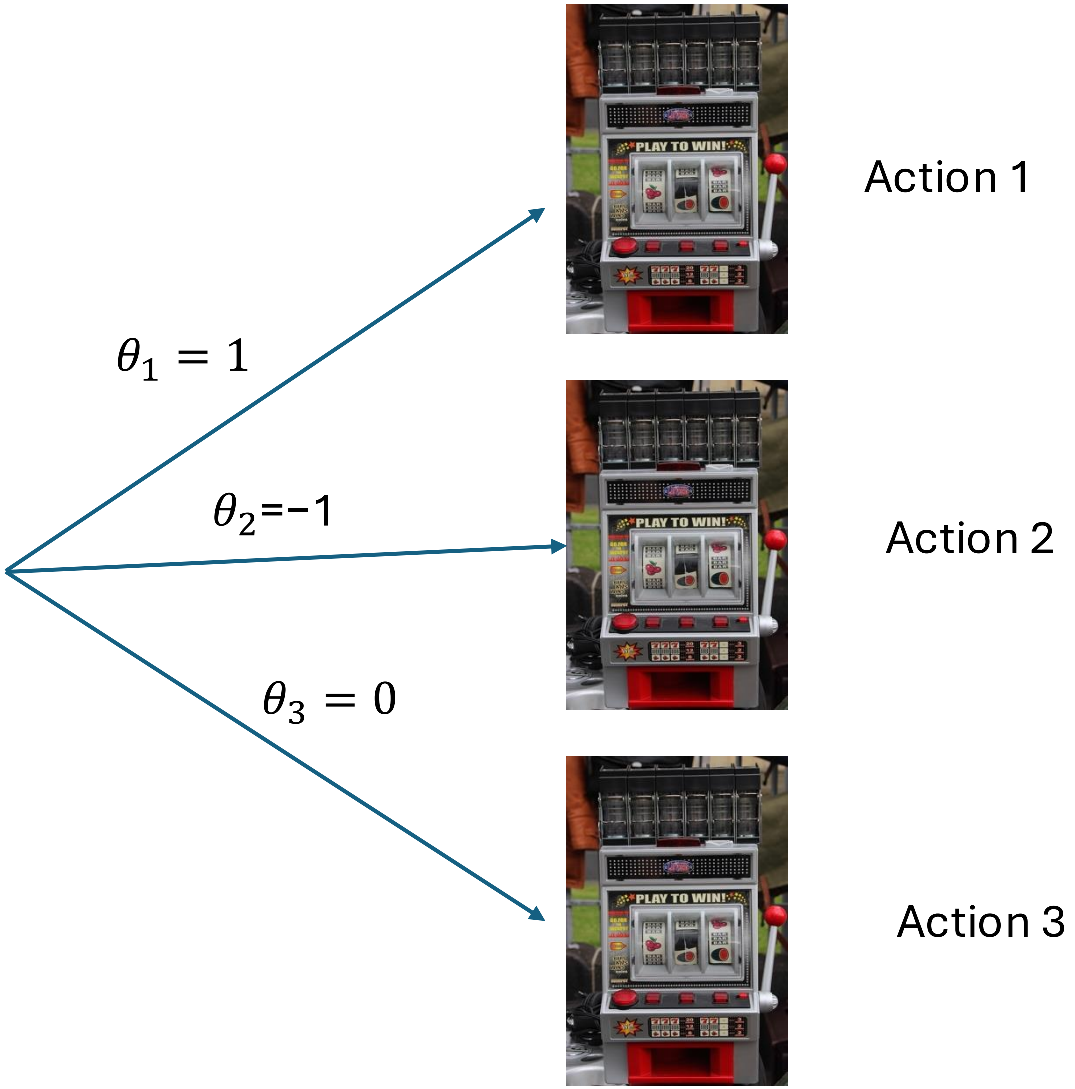

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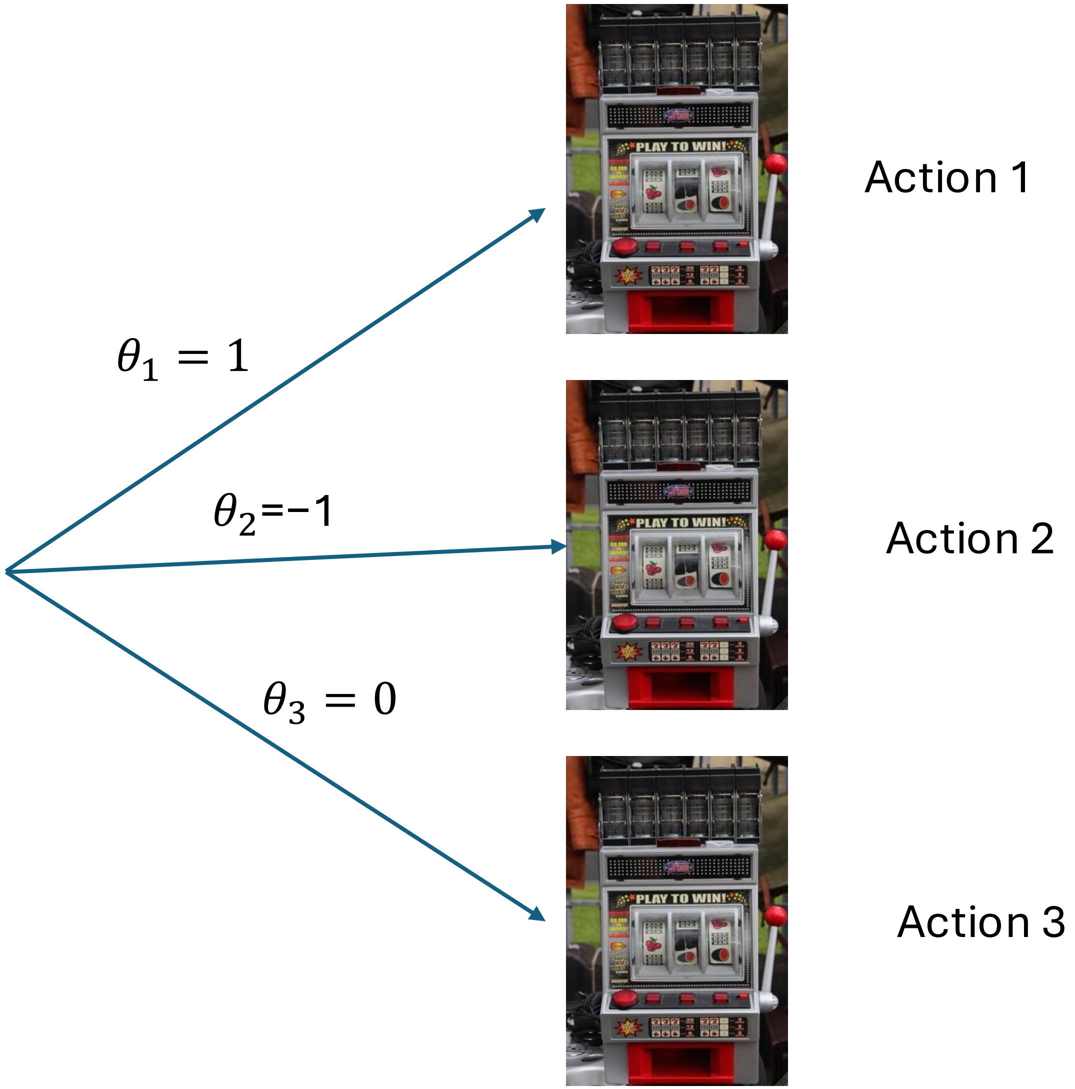
$$\pi_\theta(a_1) = \frac{e}{e + \frac{1}{e} + 1} = 0.66$$



Policy Gradient on Multi-Arm Bandits

$$\theta_1 = 1, \pi_\theta(a_1) = 0.66$$
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T

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Take 5 actions according to π_θ:

$$\tau = (a_1, 3), (a_2, -1), (a_3, 2), (a_1, 4), (a_3, 1)$$

T

Policy Gradient on Multi-Arm Bandits

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Take 1 action according to π_θ:

$$\tau = (a_1, 3)$$

Policy Gradient (without states): $\nabla_\theta J(\theta) = G_t^T \nabla_\theta \ln \pi_\theta(a \quad)$

Take Action according to Softmax:

$$\nabla_\theta J(\theta) = G_t \nabla_\theta \ln \pi_\theta(a\quad)$$

$$\pi_\theta(a_1) = \frac{e^{\theta_1}}{e^{\theta_1} + e^{\theta_2} + e^{\theta_3}}$$

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At timestep 1, a_1 got a reward of 3:

Need to compute: $G_t \nabla_\theta \ln \pi_\theta(a_1)$

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Need to compute: $G_t \nabla_\theta \ln \pi_\theta(a_1)$

What is G_0?
Bandits is not a sequential problem (no future rewards to worry about), so it's just the observed reward for taking the first action, a_1.

Take Action according to Softmax:

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$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{e^{\theta_1}}{e^{\theta_1} + e^{\theta_2} + e^{\theta_3}} \\ \cdots \\ \cdots \end{bmatrix}$$

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$$G_t \nabla_\theta \ln \pi_\theta(a_1) = 3 \cdot \begin{bmatrix} 0.34 \\ -0.09 \\ -0.25 \end{bmatrix}$$

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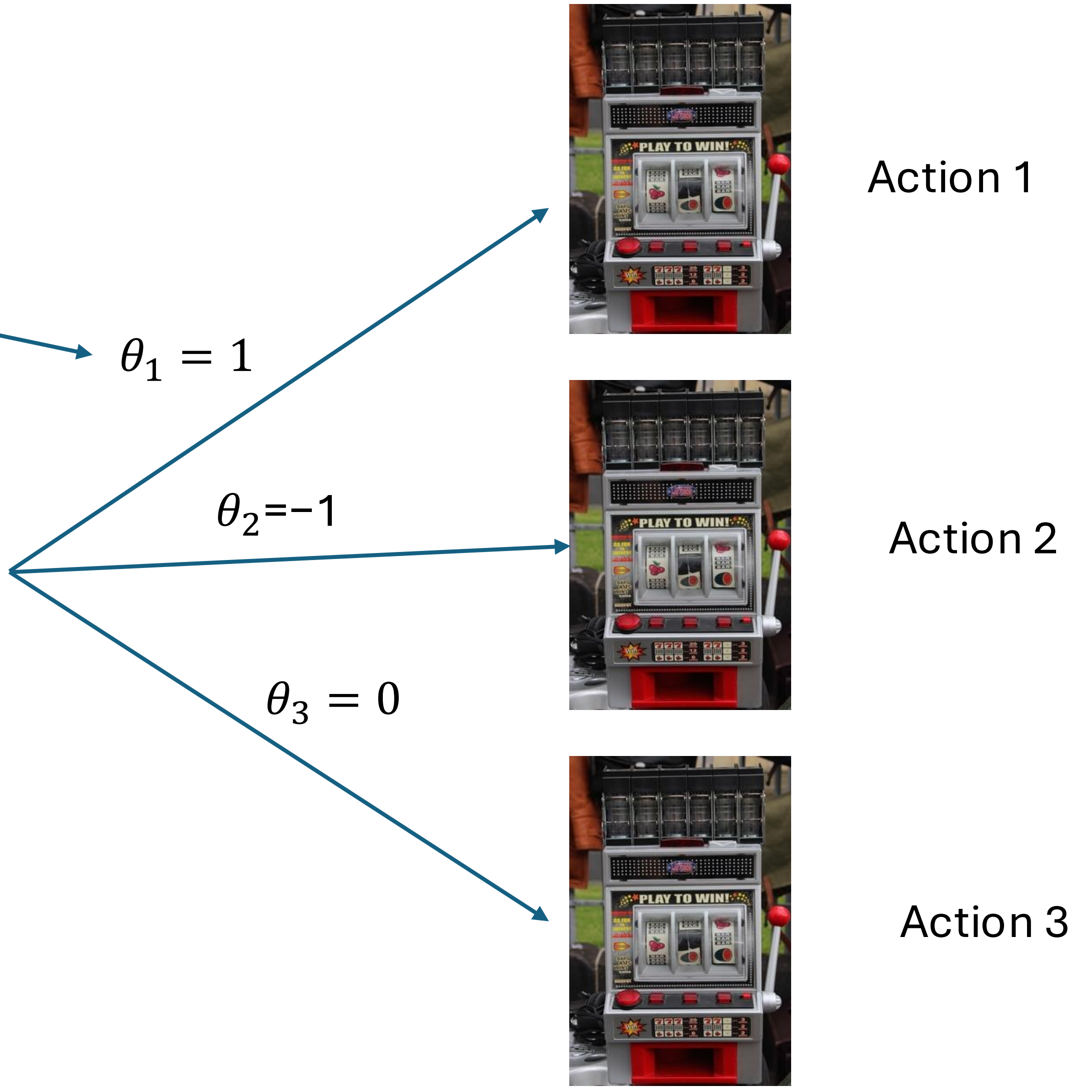
REINFORCE

What if θ is the output of a neural network?

How to compute: $G_t \nabla_\theta \ln \pi_\theta(a_t|s_t)$?



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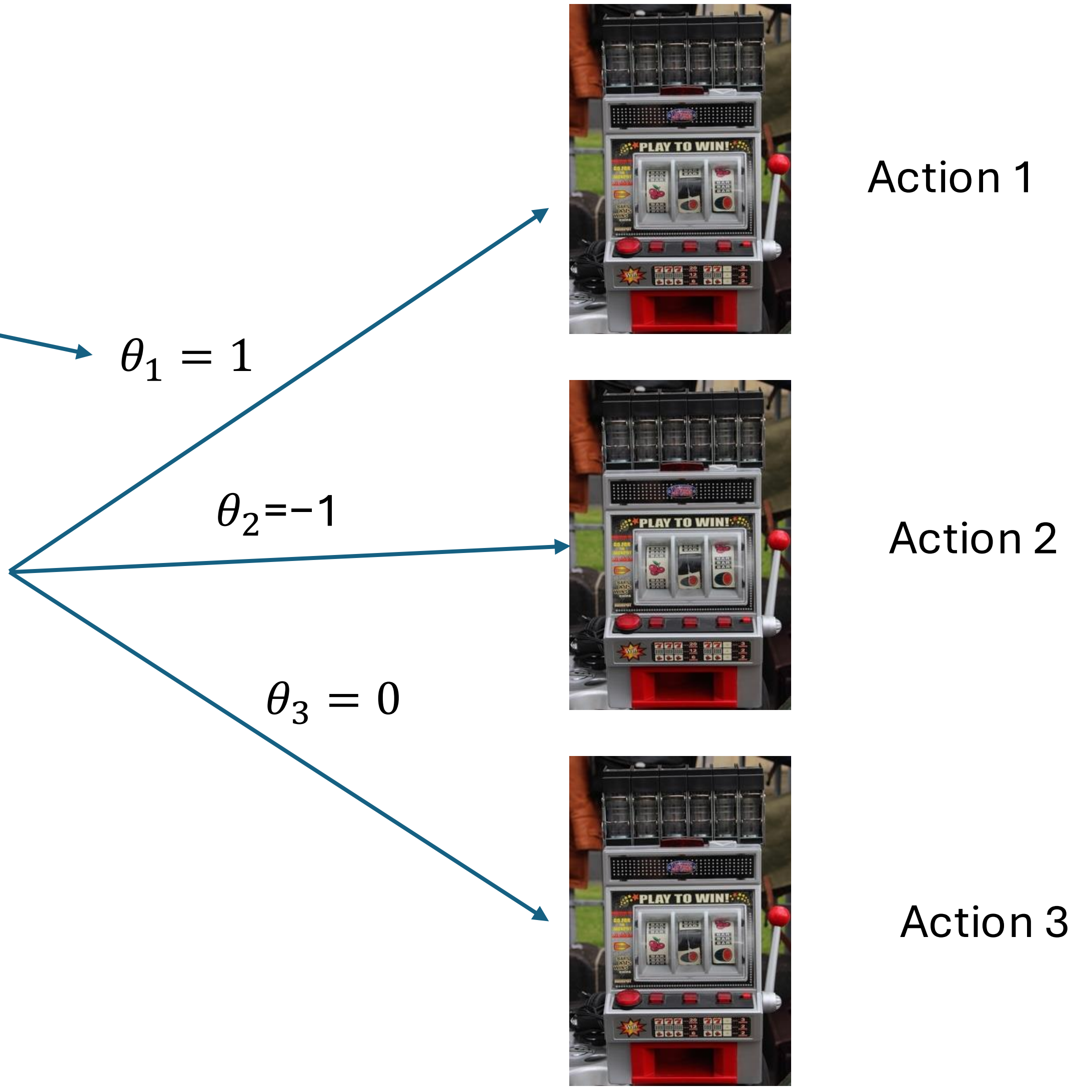

Compute $\ln \pi_\theta(a_t|s_t)$ in gradient tape context.

REINFORCE

What if θ is the output of a neural network?

How to compute:
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$\theta_1 = 1$

θ_2=−1

$\theta_3 = 0$

Action 1

Action 2

Action 3

Compute $\ln \pi_\theta(a_t|s_t)$ in gradient tape context.

But also, remember, you have to perform gradient ASCENT.
If an optimizer minimizes by default, you can use $-\nabla_\theta J(\theta)$

REINFORCE Variance

If we could calculate $\nabla_\theta J(\theta)$ exactly (not just for single trajectory/sample), then Policy Gradient would be a great algorithm! (with some minor flaws)

Results on Cartpole

Image Source: https://medium.com/@fork.tree.ai/understanding-baseline-techniques-for-reinforce-53a1e2279b57

Actor-Critic Methods

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$V(s_t) = \mathbb{E}[G_t]$ as well, but technically it's not a critic function. Critic functions critique actions.

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$$\nabla_\theta J(\theta) = \mathbb{E}[\sum_{t=0}^{T} Q^{\pi_\theta}(s_t, a_t) \nabla_\theta \ln \pi_\theta(a_t|s_t)]$$

Actor-Critic Algorithm: Learn Q^{π} and $\pi(a|s)$

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Actor (policy): Takes actions

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Actor-Critic Algorithm: Learn Q^{π} and $\pi(a|s)$

Policy network has parameters θ
Q network has parameters w

Initialize $\pi_{\theta}, Q_w, \alpha_{\theta}, \alpha_w$

Repeat forever:

Take action a, get new state s' and reward *r*

Sample next action $a' \sim \pi_{\theta}(a|s)$

update $\theta \leftarrow \theta + \alpha_{\theta} Q_w(s,a) \nabla_{\theta} \ln \pi_{\theta}(a|s)$

Calculate TD Error: $\delta = r + \gamma Q_w(s',a') - Q_w(s,a)$

update $w \leftarrow w + \alpha_w \delta \nabla_w Q_w(s,a)$

$a \leftarrow a', s \leftarrow s'$

Actor-Critic Algorithm: Learn Q^{π} and $\pi(a|s)$

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Like Q-learning and REINFORCE at the same time

Variations on a Theme...

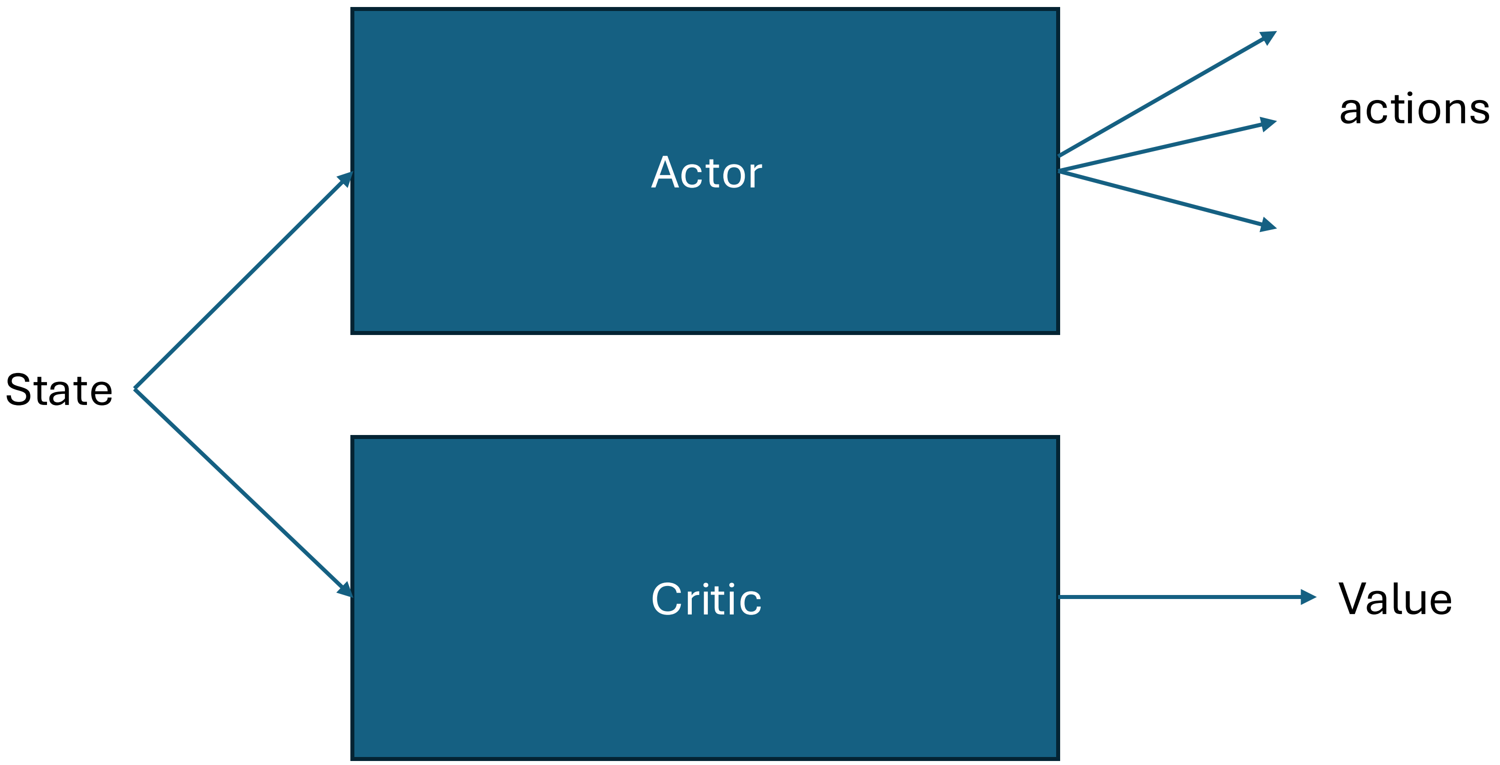
How to estimate $J(\theta)$

(Wikipedia uses R_t instead of G_t)

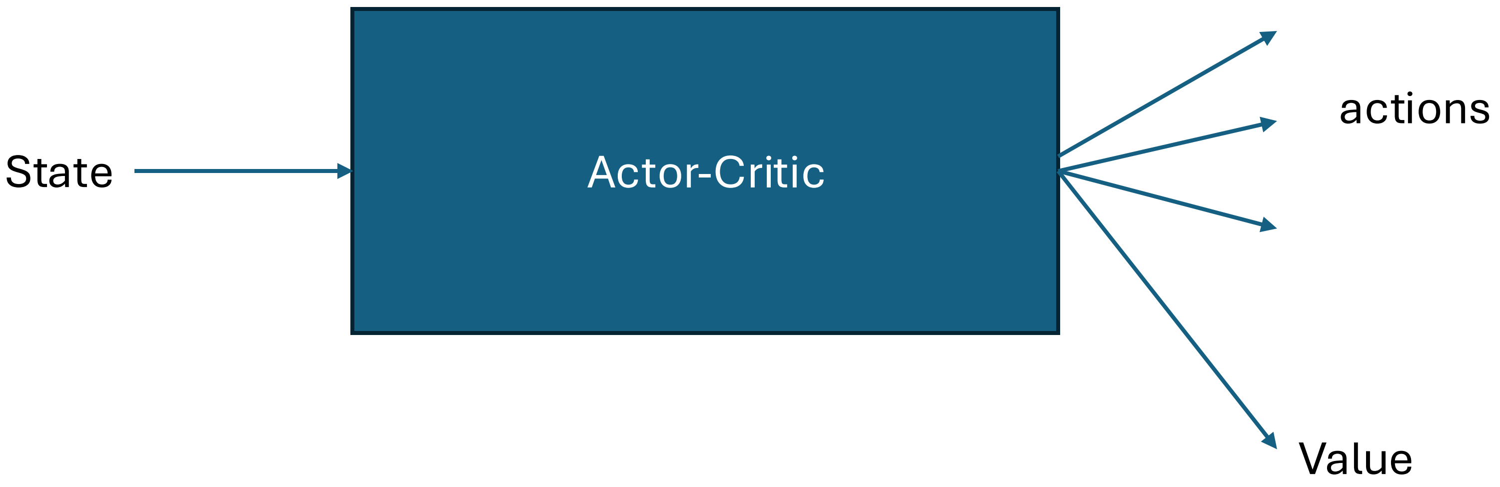
- $\sum_{0 \le i \le T} (\gamma^i R_i)$.
- $\gamma^j \sum_{j \le i \le T} (\gamma^{i-j} R_i)$: the **REINFORCE** algorithm.
- $\gamma^j \sum_{j \le i \le T} (\gamma^{i-j} R_i) - b(S_j)$: the **REINFORCE with baseline** algorithm. Here b is an arbitrary function.
- $\gamma^j \left(R_j + \gamma V^{\pi_\theta}(S_{j+1}) - V^{\pi_\theta}(S_j)\right)$: TD(1) learning.
- $\gamma^j Q^{\pi_\theta}(S_j, A_j)$.
- $\gamma^j A^{\pi_\theta}(S_j, A_j)$: **Advantage Actor-Critic (A2C)**.[3]
- $\gamma^j \left(R_j + \gamma R_{j+1} + \gamma^2 V^{\pi_\theta}(S_{j+2}) - V^{\pi_\theta}(S_j)\right)$: TD(2) learning.
- $\gamma^j \left(\sum_{k=0}^{n-1} \gamma^k R_{j+k} + \gamma^n V^{\pi_\theta}(S_{j+n}) - V^{\pi_\theta}(S_j)\right)$: TD(n) learning.
- $\gamma^j \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{1-\lambda} \cdot \left(\sum_{k=0}^{n-1} \gamma^k R_{j+k} + \gamma^n V^{\pi_\theta}(S_{j+n}) - V^{\pi_\theta}(S_j)\right)$: TD(λ) learning, also known as **GAE (generalized advantage estimate)**.[4] This is obtained by an exponentially decaying sum of the TD(n) learning terms.

Source: https://en.wikipedia.org/wiki/Actor-critic_algorithm

Actor-Critic Networks



Actor-Critic Networks



Just make sure you use the correct activation function for the different outputs

Deep Q-Learning Revisited

Compute TD-Error: $\delta = r + \gamma \max_{a'} Q(s', a') - Q(s, a)$

Loss Function: $L = \delta^2$

Update model with Gradient Descent

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Actor-Critic Uses:

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Actor-Critic Uses:

$$\delta = r + \gamma Q(s', a') \ - Q\ (s, a)$$

Q-Learning is learning *Optimal Q-values*

Actor-Critic is learning the Q-values for following a specific policy Q^{π}

On-Policy Vs. Off-Policy Learning

RL algorithms collect experiences and learn from these experiences

On-Policy Algorithms have to collect experiences with the policy they are learning

Off-Policy Algorithms can use **any** policy to collect experiences

DQNs Are Off-Policy

In Q-Learning, we typically collect experiences using an ϵ-greedy policy in training

With probability ϵ, take random action.

Else, take action $\mathrm{argmax}_a\, Q(s, a)$

At test time, we should take the best actions, not the ϵ-greedy actions

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These are different policies! DQNs can be trained with any data collection policy at training time

Actor-Critic Algorithm: Learn Q^{π} and $\pi(a|s)$

Initialize $\pi_\theta, Q_w, \alpha_\theta, \alpha_w$

Repeat forever:

Take action a, get new state s' and reward r

Sample next action $a' \sim \pi_\theta(a|s)$

On Policy: Have to take actions according to π_θ

update $\theta \leftarrow \theta + \alpha_\theta Q_w(s,a) \nabla_\theta \ln \pi_\theta(a|s)$

Calculate TD Error: $\delta = r + \gamma Q_w(s', a') - Q_w(s,a)$

update $w \leftarrow w + \alpha_w \delta \nabla_w Q_w(s,a)$

$a \leftarrow a', s \leftarrow s'$

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- More sample efficient, tend to converge faster

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Disadvantages of Off-Policy Learning:

- Slower…

For the Record: On-Policy Q-Learning (SARSA)

There is an On-Policy Q-learning algorithm:

$$\delta = r + \gamma Q(s', a') \; - Q(s, a)$$

Why is it called SARSA?

$$\delta = \gamma Q(s', a') + r \; - Q\ (s, a)$$